Mathematisches Institut
der Universität München
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WS 2001/02
Set 1

Problem set for Advanced Algebra

(1) Let R be a ring and M an abelian group. Show that there is a one-to-one correspondence between maps $f : R \times M \longrightarrow M$ that make M into a left R-module and ring homomorphisms (always preserving the unit element) $g : R \longrightarrow \mathrm{End}(M)$.

(2) Let $f : M \longrightarrow N$ be an R-module homomorphism. The following are equivalent:

(a) f is a monomorphism,

(b) for all R-modules P and all homomorphisms $g, h : P \longrightarrow M$

$$fg = fh \Longrightarrow g = h,$$

(c) for all R-modules P the homomorphism of abelian groups

$$\mathrm{Hom}_R(P, f) : \ \mathrm{Hom}_R(P, M) \ni g \mapsto fg \in \mathrm{Hom}_R(P, N)$$

is a monomorphism.

(3) Are $\{(0, \ldots, a, \ldots, 0) | a \in K_n\}$ and $\{(a, 0, \ldots, 0) | a \in K_n\}$ isomorphic as $M_n(K)$-modules?

(4) Show: $m : \mathbb{Z}/(18) \otimes_{\mathbb{Z}} \mathbb{Z}/(30) \ni \overline{x} \otimes \overline{y} \mapsto \overline{xy} \in \mathbb{Z}/(6)$ is a homomorphism and m is bijective.

Due date: Tuesday, 23.10.2001, 16:15 in Lecture Hall E06