SYMMETRIC YETTER-DRINFELD CATEGORIES ARE TRIVIAL

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We show the following theorem answering a question of S. Montgomery and D. Fischman.

Theorem . *Let H be a k-Hopf algebra such that the canonical braiding of the category $\mathcal{YD}_{\mathcal{H}}^{\mathcal{H}}$ of Yetter-Drinfeld modules is a symmetry. Then $H = k$.*

Proof. Let $M := H$ with the coregular cooperation of H as the H-comodule structure ($\delta(h) = \sum h_1 \otimes h_2$) and with the adjoint operation of H on itself as module structure ($x \cdot h = \sum S(h_1)xh_2$). Then M is a Yetter-Drinfeld module or a crossed module, i.e. a right H-module and a right H-comodule such that $\sum (x \cdot h)_0 \otimes (x \cdot h)_1 = \sum (x_0 \cdot h_2) \otimes S(h_1)x_1h_3$.

Let $N := H$ with the regular operation of H on itself as module structure ($x \cdot h = xh$) and the coadjoint cooperation of H as the H-comodule structure ($\delta(h) = \sum h_2 \otimes S(h_1)h_3$). Then N is a Yetter-Drinfeld module.

Apply the square of the braiding $\tau(x \otimes y) = \sum y_0 \otimes x \cdot y_1$ to the element $x \otimes 1 \in M \otimes N$ and use the given structures to get $x \otimes 1 = \tau^2(x \otimes 1) = \tau(1 \otimes x) = \sum x_1 \otimes x_2$ for all $x \in H$ hence $H = k$. □

Corollary . *For a finite-dimensional Hopf algebra $H \neq k$ the Drinfeld double $D(H)$ is quasitriangular, but not triangular.*

References

[M] Susan Montgomery: *Hopf Algebras and Their Actions on Rings.* CBMS 82, AMS-NSF, 1993.

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