GRAPHICAL CALCULUS PROGRAM

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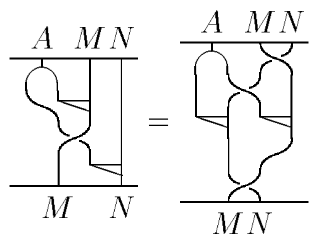
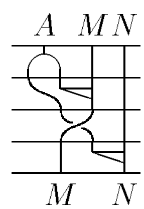
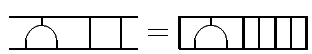
This program offers a simple way to typeset figures from *graphical calculus* as used in calculations in braided monoidal categories. Sketch an equation in graphical calculus e.g.



which represents the equation $(1_M \otimes \mu_N)(\tau^{-1} \otimes 1_N)(1_A \otimes \mu_M \otimes 1_N)(\nabla_A \otimes 1_M \otimes 1_N) = \tau^{-1}(\mu_N \otimes \mu_M)(1_A \otimes \tau \otimes 1_M)(\nabla_A \otimes \tau)$. Cut each side of the graphical equation up into single lines e.g.



and cut each line into the various boxes containing single maps e.g.

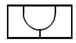
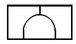
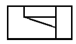
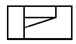
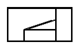
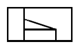
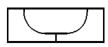
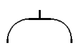
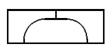
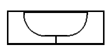
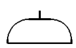
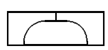
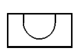
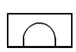
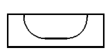
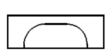
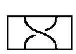
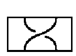
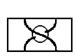


representing $\nabla_A \otimes 1_M \otimes 1_N$ and encode these maps with the symbols described below.

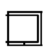
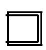
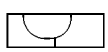
All commands for a graphical diagram start with \g .... A graphical equation is written (and centered) with the mathematical symbols $$ ... $$. Each single graphical diagram (side of an equation) is opened by \gbeg and closed by \gend. Each graphical symbol in a graphical diagram is contained in an invisible box of size x-dim by y-dim. It can have several top and bottom connectors. To interconnect the connectors of one symbol with the connectors of another symbol the command \gcon{x}{y}{t}{b} is used. The various available graphical symbols are described in the following table:

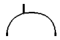
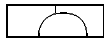
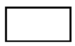
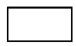
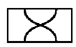
Date: 31. Oct. 1998.

name	symb.	boxed symb.	x dim	y dim	parameters	description
\gbeg			x	y	$\{x\}\{y\}$	(gbeg = graphic *beg*in) beginning of graphical diagram - parameters x and y denote the overall size of the diagram
\gend					none	(gend = graphic *end*) end of graphical diagram
\gnl					none	(gnl = graphic *n*ew *l*ine) new line command in the graphical diagram
\gvac			1	1	$\{n\}$	(gvac = graphic *vac*uous fields) empty space of width n (here $n = 1$)
\gcl	|		1	1	$\{y\}$	(gcl = graphic *c*onnecting *l*ine) connect command - vertically connects one bottom connector of one morphism with a top connector of another morphism - has vertical length y
\gcn			2	1	$\{x\}\{y\}\{t\}\{b\}$	(gcn = graphic *con*nector) connect command - connects one bottom connector of one morphism with a top connector of another morphism (see description below)
\gnot	f	$\boxed{f}$	0	1	$\{f\}$	(gnot = grphic *not*ation) specification for morphisms, to be inserted into boxed morphisms (see below)
\got	S		2	1	$\{n\}\{S\}$	(got = graphic *o*bject *t*op) specification for the domain of morphisms, to be used at the top of a graphical diagram - 1st parameter is width n of box, 2nd parameter is specification for the object S
\gob	S		2	1	$\{n\}\{S\}$	(gob = graphic *o*bject *b*ottom) specification for the range of morphisms, to be used at the bottom of a graphical diagram

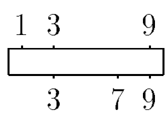
name	symb.	boxed symb.	x dim	y dim	parameters	description
\gmu			2	1	none	(gmu = graphic *mu*ltiplication) multiplication *)
\gcmu			2	1	none	(gcmu = graphic *comu*ltiplication) comultiplication *)
\glm			2	1	none	(glm = graphic *l*eft *m*odule) left module action
\grm			2	1	none	(grm = graphic *r*ight *m*odule) right module action
\glcm			2	1	none	(glcm = graphic *l*eft *c*o-*m*odule) left comodule coaction
\grcm			2	1	none	(grcm = graphic *r*ight *c*o-*m*odule) right comodule coaction
\gwmu			3	1	$\{n\}$	(gwmu = graphic *w*ide *mu*ltiplication) wide multiplication - width n (here $n = 3$) is given by the parameter *)
\gwcm			3	1	$\{n\}$	(gwcm = graphic *w*ide *c*o-*m*ultiplication) wide comultiplication *)
\gwmuc			3	1	$\{n\}$	(gwmuc = graphic *w*ide *mu*ltiplication *c*losed) wide closed multiplication symbol *)
\gwcmc			3	1	$\{n\}$	(gwcmc = graphic *w*ide *c*o*m*ultiplication *c*losed) wide closed comultiplication symbol *)
\gev			2	1	none	(gev = graphic *ev*aluation) evaluation
\gdb			2	1	none	(gdb = graphic *d*ual *b*asis) dual basis
\gwev			3	1	$\{n\}$	(gwev = graphic *w*ide *ev*aluation) wide evaluation
\gwdb			3	1	$\{n\}$	(gwdb = graphic *w*ide *d*ual *b*asis) wide dual basis
\gbr			2	1	none	(gbr = graphic *br*aid) braid
\gibr			2	1	none	(gibr = graphic *i*nverse *br*aid) inverse braid
\gbrc			2	1	none	(gbrc = graphic *br*aid *c*ircled) circled braid

name	symb.	boxed symb.	x dim	y dim	parameters	description
\gibrc			2	1	none	(gibrc = graphic inverse braid circled) inverse circled braid
\gu			1	1	$\{n\}$	(gu = graphic unit) unit *)
\gcu			1	1	$\{n\}$	(gcu = graphic co-unit) counit *)
\gmp			1	1	$\{S\}$	(gmp = graphic morphism) morphism - specification S given by the parameter
\gbmp			1	1	$\{S\}$	(gbmp = graphic boxed morphism) boxed morphism - specification S given by the parameter
\glmptb			1	1	none	(glmptb = graphic left [part of] morphism [with] top bottom [connectors]) left part of long boxed morphism - specification S can be inserted with \gnot - has top and bottom connectors
\glmpt			1	1	none	(glmpt = graphic left [part of] morphism [with] top [connector]) left part of long boxed morphism - has only top connector
\glmpb			1	1	none	(glmpb = graphic left [part of] morphism [with] bottom [connector]) left part of long boxed morphism - has only bottom connector
\glmp			1	1	none	(glmp = graphic left [part of] morphism) left part of long boxed morphism - has no connectors
\gcmptb			1	1	none	(gcmptb = graphic center [part of] morphism [with] top bottom [connectors]) center of long boxed morphism - specification S can be inserted with \gnot - has top and bottom connectors
\gcmpt			1	1	none	(gcmpt = graphic center [part of] morphism [with] top [connector]) center of long boxed morphism - has only top connector

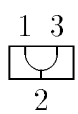
name	symb.	boxed symb.	x dim	y dim	parameters	description
\gcmpb			1	1	none	(gcmpb = graphic *c*enter [part of] *m*or*p*hism [with] *b*ottom [connector]) center of long boxed morphism - has only bottom connector
\gcmp			1	1	none	(gcmp = graphic *c*enter [part of] *m*or*p*hism) center of long boxed morphism - has no connectors
\grmptb			1	1	none	(grmptb = graphic *r*ight [part of] *m*or*p*hism [with] *t*op *b*ottom [connectors]) right part of long boxed morphism - specification S can be inserted with \gnot - has top and bottom connectors
\grmpt			1	1	none	(grmpt = graphic *r*ight [part of] *m*or*p*hism [with] *t*op [connector]) right part of long boxed morphism - has only top connector
\grmpb			1	1	none	(grmpb = graphic *r*ight [part of] *m*or*p*hism [with] *b*ottom [connector]) right part of long boxed morphism - has only bottom connector
\grmp			1	1	none	(grmp = graphic *r*ight [part of] *m*or*p*hism) right part of long boxed morphism - has no connectors
\gwmuh			3	1	$\{n\}\{l\}\{r\}$	(gwmuh = graphic *w*ide *mu*ltiplication [with] *h*alf [length count for connectors]) wide multiplication with top connectors at positions l and r (here $l = 1$, $r = 4$) - width n (here $n = 3$) - bottom connector is in the middle of the box (here at position 3) *)

name	symb.	boxed symb.	x dim	y dim	parameters	description
\gwcmh			3	1	$\{n\}\{l\}\{r\}$	(gwcmh = graphic *w*ide *co*-*m*ultiplication [with] *h*alf [length count for connectors]) wide comultiplication with bottom connectors at positions l and r (here $l = 2$, $r = 5$) - width n (here $n = 3$) - top connector is in the middle of the box (here at position 3) *)
\gsbox			2	1	$\{n\}$	(gsbox = graphic *s*ymbol *box*) box around other symbols as used in this table - width n (here $n = 2$)
\gsy			2	1	none	(gsy = graphic *sy*mmetry) symmetry

Most of the symbols have one or several connectors at the top and at the bottom. They stand for the number of tensor factors in the domain and the range of the corresponding morphism. So the multiplication of an algebra $\nabla : A \otimes A \to A$ with the corresponding graphical diagram  has two connectors on top and one connector at the bottom. These must be connected with other connectors (composition of morphisms). Most connectors appear in the imaginary boxes for the graphical symbols



at odd positions. There are some morphisms (denoted by *)) that have connectors at even positions for example



We count the positions of the possible connectors along the (imaginary) box in which the morphism is positioned from 0 to 2×length of the box, so for the multiplication box (2×1) from 0 to 4. These connectors must be connected to other connectors. For this purpose we have the connection symbol \gcn$\{x\}\{y\}\{t\}\{b\}$ with 4 parameters. This connector sits in a box of its own. This is one of two boxes that can have y-size bigger than 1, so it can hang below the line (in which it is defined). The x-parameter measures the width of the box. This width moves the following morphism to the right. The y-parameter measures the height of the box. If $y > 1$ then the box extends below the present line. (It will never extend above.) The connector connects from the top position t (from a bottom connector of another symbol) to the bottom position b

(leading to a top connector of another symbol). The top and bottom positions t and b are defined with respect to the box of the connector as described above. (They may be negative.) The overhanging box does not disturb the lower lines.

Since many of such connectors are needed in complicated diagrams and since they are graphically very complicated we also have a very simple connector \gcl{y} consisting just of a vertical line connecting position 1 to position 1. The following commands give the same result:

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\gcl{y}
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and

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\gcn{1}{y}{1}{1}.
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The first command should be preferred because the dvi-page built by it is much less complicated.

The original graphical equation on the first page is written as

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$$\gbeg{4}{6}
\got{2}{A} \got{1}{M} \got{1}{N} \gnl
\gcmu \gcl{1} \gcl{3} \gnl
\gcn{1}{1}{1}{3} \glm \gnl
\gvac{1} \gibr \gnl
\gcn{2}{1}{3}{3} \glm \gnl
\gob{3}{M} \gob{1}{N}
\gend =
\gbeg{4}{7}
\got{2}{A} \got{1}{M} \got{1}{N} \gnl
\gcmu \gbr \gnl
\gcl{1} \gbr \gcl{1} \gnl
\glm \glm \gnl
\gcn{2}{1}{3}{3} \gcn{2}{1}{3}{1} \gnl
\gvac{1} \gibr \gnl
\gvac{1} \gob{1}{M} \gob{1}{N}
\gend $$
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