Conference on „Recent Results on Quantum Many-Body Systems“

Titles and Abstracts

Volker Bach, The Witten Laplacian and Infrared Bounds

Abstract: Almost five decades ago, reflection positivity was discovered and used as a powerful tool to derive infrared bounds for ferromagnetic classical statistical mechanical lattice systems with a continuous symmetry that imply the existence of a phase transition at sufficiently low temperature in $d \geq 3$ spatial dimensions. We also establish such infrared bounds for ferromagetic systems, but without using reflection positivity nor assuming translation invariance. Instead, our derivation uses the Witten Laplacian and the Helffer-Sj{\"o}strand Formula. This is joint work with Fiona Gottschalk.

Rafael Benguria, Gagliardo-Nirenberg-Sobolev inequalities for convex domains in $\R^d$

Abstract: A special type of Gagliardo--Nirenberg--Sobolev (GNS) inequalities in $\mathbb{R}^d$ has played a key role in several proofs of Lieb--Thirring inequalities. Recently, the need for GNS inequalities in convex domains of $\mathbb{R}^d$, in particular for cubes, has arised. In this talk I will present a proof of a GNS inequality for convex domains, with explicit constants which depend on the geometry of the domain. Later, using the discrete version of Rumin's method, I give a proof of GNS inequalities on cubes with improved constants. This is joint work with Cristobal Vallejos (PUC) and Hanne Van Den Bosch (Universidad de Chile)

Li Chen, The Maximal Negative Ion of the Time-Dependent Thomas-Fermi and the Vlasov Atom

Abstract: We show an atom of atomic number Z described by the time-dependent Thomas-Fermi equation or the Vlasov equation cannot bind more than $4Z$ electrons.

László Erdös, Matrix Dyson equation in random matrix theory

Abstract: We discuss the fundamental properties of the matrix Dyson equation arising in the theory of random matrices with correlated entries. In particular, we show that the density of states is an analytic function with possibly square root and cubic cusp singularities at the spectral edges.

Marcel Griesemer, Spectral Theory of the Fermi Polaron

Abstract: The Fermi-polaron refers to a system of free fermions interacting with an impurity particle by means of two-body contact forces. I will present a general mathematical framework for defining many-body Hamiltonians with two-body contact interactions. A novel variational principle, established within this general framework, allows us to link the low-lying eigenvalues of the Fermi-polaron in a 2-dimensional box to the zero-modes of a BirmanSchwinger type operator. This allows us to show, e.g., that the polaron- and molecule energies, as computed in the physical literature, are indeed upper bounds to the ground state energy of system. - This is joint work with Ulrich Linden.

Christian Hainzl, The interacting Fermi gas: A step beyond Hartree-Fock

Abstract: We consider a homogenous Fermi gas interacting via a regular mean-field potential. We recover the correlation energy to leading order in the coupling parameter.

Bernard Helffer, On the extended Courant nodal property: examples and conterexamples

Abstract: In a footnote of Courant-Hilbert, it is claimed that Courant nodal theorem extends to linear combination of eigenfunctions. This statement is correct in dimension 1 (Theorem of Sturm) but as observed by V. Arnold in the seventies would imply wrong statements in Algebraic Geometry. In this talk, we would like to present very simple examples contradicting Courant-Hilbert's statement and discuss alternative statements. This is a work in collaboration with P. Bérard.

Dirk Hundertmark, Cwikel's bound reloaded

Abstract: There are a couple of proofs by now for the famous Cwikel Lieb Rozenblum (CLR) bound, which is a semiclassical bound on the number of bound states for a Schr\"odinger operator, proven in the 1970s.
Of the rather distinct proofs by Cwikel, Lieb, and Rozenblum, the one by Lieb yields the best constant, the one by Rozenblum does not seem to yield any good or even reasonable estimate for the constants, and Cwikel's proof is said to yield a constant which is at least about 2 orders of magnitude off the truth. This situation did not change much during the last 40+ years, even though there were other proofs of the CLR inequality later, the most radical one based on ideas of Rumin in the work by Rupert Frank.

It turns out that this common belief, i.e, Cwikel's approach yields bad constants, is not set in stone: We give a drastic simplification of Cwikel's original approach which leads to a rather good bound for the constant in the CLR inequality. Moreover, our proof highlights a natural but overlooked connection of the CLR bound with maximal Fourier multiplier estimates from harmonic analysis. This is joint work with Peer Kunstmann, Tobias Ried, and Semjon Wugalter.

Mathieu Lewin, A compactness theorem for chemical reactions in quantum mechanics

Abstract:
In the Born-Oppenheimer approximation, an isomerization is a path linking two local minima of the ground state energy of a molecule, when the nuclear positions are varied. In this talk I will discuss the plausible conjecture that all isomerizations are compact for neutral molecules. That is, a sequence of paths approaching the lowest possible activation energy must only involve nuclei staying in a compact set. Passing to the limit, one then obtains a critical point at the mountain pass level, interpreted as a transition state in chemistry. I will review some results on this problem, including a recent work with Ioannis Anapolitanos (KIT). Work supported by ERC grant MDFT.

Elliott Lieb, Proof of a Conjecture of Carbery

Abstract: Consider the L^p triangle inequality for functions, |f+g| \leq |f|+|g|, which is saturated when f=g, but which is poor when f and g have disjoint support. Carbery proposed a slightly more complicated inequality to take into account the orthogonality, or lack of it, of the two functions. With Eric Carlen and Rupert Frank a strengthened version of the conjecture has now been proved. Actually, Carbery was mainly interested in (noncommutative) matrices and traces instead of functions and integrals, so there is still much to be done.

Michael Loss, A Quantum Kac Model

Abstract: We introduce a quantum version of the Kac Master equation, and explain issues like equilibria, propagation of chaos and the the corresponding quantum Boltzmann equation. This is joint work with Eric Carlen and Maria Carvalho.

Benjamin Schlein, Excitation spectrum of Bose-Einstein condensates

Abstract: We consider systems of N bosons confined in a box with volume one and interacting through a potential with scattering length of the order 1/N (Gross-Pitaevskii regime). For non-negative and sufficiently weak interactions, we determine the low-energy spectrum, i.e. the ground state energy and low-lying excitations, up to errors that vanishes in the limit of large N, confirming the validity of Bogoliubov's predictions.

Robert Seiringer, Bose-Einstein Condensation in a Dilute, Trapped Gas at Positive Temperature

Abstract: We consider an interacting, dilute Bose gas trapped in a harmonic potential at positive temperature, in a combination of a thermodynamic and a Gross-Pitaevskii (GP) limit. We prove that the difference between the canonical free energy of the interacting gas and the one of the noninteracting system can be obtained by minimizing the GP energy functional. We also prove Bose-Einstein condensation, in the sense that the one-particle density matrix of the interacting Gibbs state is to leading order given by that of the noninteracting gas but with the free condensate wavefunction replaced by the GP minimizer. (Joint work with A. Deuchert and J. Yngvason)

Jan Philip Solovej, Universality of Born-Oppenheimer curves for diatomic molecules

Abstract: I will discuss Born-Oppenheimer curves for diatomic molecules in the reduced Hartree-Fock model (Hartree-Fock without the exchange term). I will discuss the limit of large molecules (large Z). In this limit the Born-Oppenheimer curves will approach a universal short distance asymptotic behavior given by Thomas-Fermi Theory. It is worth emphasizing that we are not rescaling distances in the large Z limit. Hence short distances are large on the Thomas-Fermi scale $Z^{-1/3}$. Finally, I will compare some numerical calculations of the Thomas-Fermi curve with quantum chemistry calculations of Hartree-Fock curves. The theoretical work on reduced Hartree-Fock and the numerics of Thomas-Fermi theory is joint with Anton Samojlow. The quantum chemistry calculations are joint with Gilka and Taylor.

Edgardo Stockmeyer, Asymptotic dynamics for certain 2-D magnetic quantum systems

Abstract: In this talk I will present new results concerning the long time localisation in space (dynamical localisation) of certain two-dimensional magnetic quantum systems. The underlying Hamiltonian may have the form $H=H_0+W$, where H_0 has dense point spectrum and rotational symmetry and W is a perturbation that breaks the symmetry. (Joint with: I. Anapolitanos, E. Cardenas, D. Hundertmark, and S. Wugalter)

Timo Weidl, Edge resonances in elastic media with zero Poisson coefficient

Abstract: A two-dimensional elastic semistrip with stress-free boundary conditions and zero Poisson coefficients has an embedded eigenvalue on top of the continuous spectrum. This effect is known as edge resonance. For an infinite plate of finite thickness with a drilling hole $(R^{2}\setminus\Omega)\times I$ actually infinitely many edge resonances will occur. This is related to the spectral problem of perturbations of symbols with strongly degenerated minima, which also appear in BCS theory. Recently new methods for dealing with perturbations by boundary conditions (cracks) have been developed. I give an overview on some of our results in this area, which still poses a number of mathematical challenges.

Rudi Weikard, Spectral Theory for Systems of Ordinary Differential Equations with Distributional Coefficients

Abstract: We discuss the spectral theory of the first-order system $Ju'+qu=wf$ of differential equations on the real interval (a,b) when J is a constant, invertible skew-Hermitian matrix and q and w are matrices whose entries are distributions of order zero with q Hermitian and w non-negative. We do not require the definiteness condition customarily made on the coefficients of the equation.
Specifically, we construct associated minimal and maximal relations, and study self-adjoint restrictions of the maximal relation. For these we construct Green's function and prove the existence of a spectral (or generalized Fourier) transformation.

Semjon Wugalter, Van der Waals Interactions of Heavy Atoms

Abstract: The Van der Waals force between atoms and molecules plays an important role in chemistry, physics and biology. It explains di_erent processes and phenomena from condensation of water up to shapes of gigantic molecules such as proteins and DNA. Mathematically rigorous computation of the Van der Waals interaction energy was given recently by I.Anapolitanos and I.M.Sigal for non-relativistic Schrödinger operators. In the talk, applying a di_erent method, this result will be extended to operators with pseudorelativistic kinetic energy. In addition to this extension in both relativistic and non-relativistic cases we compute higher order corrections to the Van der Waals-London law.
The talk is based on a joint work with J.-M.Barbaroux, M.Hartig and D.Hundertmark.

Kenji Yajima, Threshold singularities and L^p-bounded of wave operators for two dimensional Schrödinger operators with point interactions

Abstract: We study the threshold behavior of the resolvent of two dimensional Schrödinger operators with point interactions. We show there are four types of behavior, regular type, resonances of s-wave and p-wave types and zero energy eigenvalue. We show that wave operators are bounded in L^p for all $1<p<\infty$ if it is of regular type. This is a joint work with Horia Cornean and Alessandro Michelangeli.