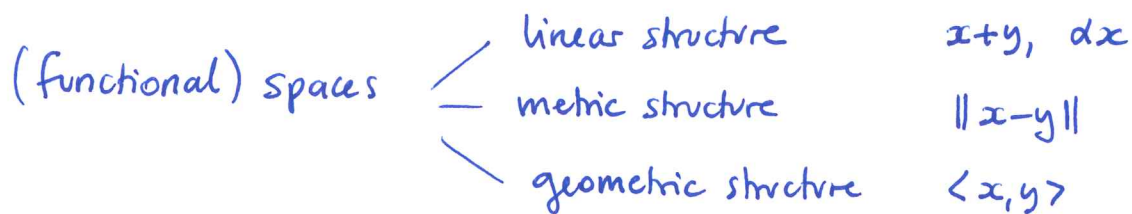


# FUNCTIONAL ANALYSIS



- topological spaces: no quant. analysis of approximations, only qualitative statements about eventual convergence
- metric spaces: quantitative concept of closeness (distance)
- normed (linear) spaces: algebraic structure + metric structure compatible with vector space operations  
( $\|x+y\| \leq \|x\| + \|y\|$ ;  $\|\alpha x\| = |\alpha| \|x\|$ )
- Banach spaces: completeness!
- Hilbert spaces: scalar product ("geometry")

## BANACH SPACE framework

Uniform boundedness theorem (Banach-Steinhaus)

⇓

Open mapping thm

⇓

Inverse mapping thm

⇓

Closed graph thm

## HILBERT SPACE framework

- Projection thm  $\mathcal{H} = \bar{M} \oplus M^\perp$
- Riesz lemma  $\phi \in \mathcal{H}^*$ ,  $\phi = \langle y_\phi, \cdot \rangle$
- Parallelogram law  
 $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$
- Polarisation