FUNCTIONAL ANALYSIS

(functional) spaces
- linear structure $x+y$, αx
- metric structure $\|x-y\|$
- geometric structure $\langle x,y\rangle$

- topological spaces : no quant. analysis of approximations, only qualitative statements about eventual convergence
- metric spaces : quantitative concept of closeness (distance)
- normed (linear) spaces : algebraic structure + metric structure compatible with vectorspace operations ($\|x+y\| \le \|x\| + \|y\|$; $\|\alpha x\| = |\alpha|\,\|x\|$)
- Banach spaces : completeness!
- Hilbert spaces : scalar product ("geometry")

BANACH SPACE framework

Uniform boundedness theorem (Banach-Steinhaus)

$\Downarrow$

Open mapping thm

$\Downarrow$

Inverse mapping thm

$\Downarrow$

Closed graph thm

HILBERT SPACE framework

- Projection thm $\mathcal{H} = \overline{\mathcal{M}} \oplus \mathcal{M}^\perp$
- Riesz lemma $\phi \in \mathcal{H}^*$, $\phi = \langle y_\phi, \cdot\rangle$
- Parallelogram law
  $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$
- Polarisation