Welcome to

Random Walks, Random Graphs and Random Media

A Workshop in Munich, September 9-13, 2019.

Scientific program.

	Monday	Tuesday	Wednesday	Thursday	Friday	
9:00–10:00	Sturm	Gurel-G	Peled	Sturm	9:00–10:00	Peled
10:15–11:15	Gurel-G	Peled	Sturm	Gurel-G	10:05–10:50	Crawford
11:30–12:15	Hanson	Guerra	Deuschel	Amir	11:15–12:00	Rodriguez
					12:05–12:50	Wachtel
14:00–14:45	Kious	Guo	—	Rosenthal		
14:50–15:35	Holmes	Deijfen	—	Procaccia		End of
16:00–16:45	Sapozhnikov	Černý	—	Mukherjee		workshop.
16:50–17:35	Biskup	Addario-B	—	Mörters		Good bye
17:35–18:30	*wine&cheese*					
18:30			Goldschmidt			

Location. The talk of Christina Goldschmidt on Wednesday evening is held at LMU's *Center for Advanced Studies*, Seestraße 13, 80802 Munich. Mind that this is at 40 minute walking distance from the Mathematical Institute (see map enclosed).

All other talks take place at the Mathematical Institute of LMU Munich (Theresienstr. 39) at the Oskar-Perron-Hörsaal.

Thanks. The organizers are most grateful for the assistance of Carola Schmaus, Daniel Willhalm, Dominik Schmid, Kilian Matzke, Leon Ramzews, Leonid Kolesnikov, Michaela Platting, Matija Pasch, Nannan Hao, Simon Reisser, Thomas Beekenkamp and Wyonie Dammann, who all worked hard in the preparation of this meeting.

We wish you an inspiring and fruitful workshop week in Munich!

Noam Berger, Alexander Drewitz, Nina Gantert, Markus Heydenreich, Alejandro Ramírez

Mini-courses

Anja Sturm: **Duality of interacting particle systems and recursive tree processes connected to mean-field limits**

In this minicourse we consider interacting particle systems and their description via graphical respresentations (stochastic flows). Based on this we examine general methods for constructing (pathwise) dual processes if the particle systems are monotone or additive. Loosely speaking, the dualities originate from considering the (potential) genealogy of a given or a set of configurations. In the additive case the system and its dual can be shown to have a percolation structure.

Ori Gurel-Gurevich: **Random walks on circle packings of planar graphs**

We will discuss several results relating the behavior of a random walk on a planar graph and the geometric properties of a nice embedding of the graph in the plane (specifically, circle packing of the graph). For example, if the graph has bounded degrees, the simple random walk is recurrent if and only if the boundary of the nice embedding is a polar set (that is, Brownian motion misses it almost surely). If the degrees are unbounded, this is no longer true, but for the case of circle packing of a triangulation, there are weights which are obtained naturally from the circle packing, such that when the boundary is polar, the weighted random walk is recurrent (we believe the converse also hold). These weights arise also in the context of discrete holomorphic and harmonic functions, a discrete analog of complex holomorphic functions. As the sizes of circles, or more generally, the lengths of edges in the nice embedding tend to zero, the discrete harmonic functions converge to their continuous counterpart with the same boundary conditions. Equivalently, the exit measure of the weighted random walk converges to the exit measure of standard Brownian motion. One of the main tools is the electric network theory of random walks. We recommend the participants be familiar with it. A useful reference for this and some of the results we will discuss is Asaf Nachmias' Saint-Flour lecture notes, which can be found at `https://arxiv.org/abs/1812.11224`.

Ron Peled: **Proper colorings of the Z^d lattice**

Consider the task of coloring the vertices of a large discrete cube in the integer lattice Z^d with q colors so that no two adjacent vertices are colored the same. In how many ways can this be done? Sampling such a coloring uniformly at random, does it exhibit any large-scale structure? Does it have fast decay of correlations? We discuss these questions and the way their answers depend on the dimension d and the number of colors q. Motivations are provided from statistical physics (anti-ferromagnetic materials, square ice), combinatorics (proper colorings, independent sets) and the study of random Lipschitz functions on a lattice. The discussion will introduce a diverse set of tools, useful for this purpose and for other problems, including entropy and coupling methods, Gibbs measures and their classification and refined contour analysis.

Talks

Louigi Addario-Berry: **Hipster random walks and their ilk**

I will describe how certain recursive distributional equations can be solved by importing rigorous results on the convergence of approximation schemes for degenerate PDEs, from numerical analysis. This project is joint work with Luc Devroye, Celine Kerriou, and Rivka Maclaine Mitchell.

Gidi Amir: **Majority dynamics: Some old and new conjectures**

In majority dynamics on a graph $G = (V, E)$, every vertex initially holds an opinion in $\{-1, 1\}$, and the opinion evolve in time according to the "majority" rule: When a vertex "rings" (e.g. according to a Poisson clock attached to each vertex) it adopts the opinion of the majority of its neighbors. The model turns out to exhibit interesting behavior on both finite and infinite graphs. Among the natural questions one can ask are questions regarding fixation and convergence of the opinions (Does every vertex eventually fixate on an opinion, if so, what can be said on the final configurations, and if not, what are the stationary measures of the process), Percolation type questions (will an infinite connected component appear or disappear with time), and questions regarding the probability of having an opinion of "1" at time t given that we started with i.i.d. Bernoulli(p) opinions at time 0. We will survey some old a new results on these and related questions, with an emphasis on some conjectures I find particularly interesting. We will also discuss a related model (Median dynamics/process, see `https://arxiv.org/abs/1904.11625`) which offers a continuous-opinion generalization of majority dynamics, and show how it offers a good language for some of the above conjectures as well as raises new questions of its own. The talk is based on joint works with Rangel Baldasso and Nissan Bailin.

Marek Biskup: **Exceptional sets of two-dimensional random walks in planar domains**

I will consider the simple random walk on planar graphs that are scaled up versions of nice continuum (planar) domains with wired boundary condition. For the walk observed at a time proportional to the cover time, I will describe the scaling limit of several natural exceptional sets of the local time — namely, the so called thick, thin, light and avoided points. It turns out that these are closely related to the thick points of the two-dimensional Gaussian Free Field in these domains and their limit law is thus captured by the various instances of the Liouville Quantum Gravity measure in the continuum domain. Based on joint work with Yoshihiro Abe and Sangchul Lee.

Jiří Černý: **Level-set percolation of the Gaussian free field on regular graphs and trees.**

We study the behaviour of level sets of 0-mean Gaussian free field on regular expanding graphs, and (at least partially) prove that they exhibit a similar phase transition as Bernoulli percolation and the vacant set of random walk on such graphs. The main ingredient is a comparison to level sets of Gaussian free field on regular trees, whose properties we describe in detail. This is a joint work with A. Abächerli (ETHZ)

Nicholas Crawford: **Hyperbolic Nonlinear Sigma models and Spanning Forest Measures**

In this talk we detail emerging connections between the statistical mechanics associated nonlinear sigma models of hyperbolic type (probability measures on the space of maps from Z^d into a hyperbolic target space) and the statistical mechanics of unrooted spanning forests. This connection may be viewed as an analog of the connection between vertex reinforced jump processes discovered by Sabot and Tarres (2011) and fully explained by Bauerschmidt, Helmuth and Swan (2017). What makes this connection particularly interesting is that understanding properties on each side of this connection is crucial to proving concrete results the other. Based on joint works with Bauerschmidt, Helmuth and Swan.

Mia Deijfen: **Competing frogs on Z^d**

The so called frog model on Z^d is driven by moving particles on the sites of the Z^d-lattice. Each site is independently assigned a random number of particles. At time 0, the particles at the origin are activated, while all other particles are sleeping. When a particle is activated, it starts moving according to a simple random walk and, when a site is visited by an active particle, any sleeping particles at the site are activated and start moving. I will describe a two-type version of the model, where an active particle can be of either of two types. For this model, a natural question is whether the types can coexist in the sense that they both activate infinitely many particles. I will describe existing results and open problems related to this.

Jean-Dominique Deuschel : **Harnack inequality for degenerate balanced random walks**

We consider an i.i.d. balanced environment $\omega(x,e) = \omega(x,-e)$, genuinely d dimensional on the lattice and show that there exist a positive constant C and a random radius $R(\omega)$ with streched exponential tail such that every non negative ω harmonic function u on the ball B_{2r} of radius $2r > R(\omega)$, we have $\max_{B_r} u <= C \min_{B_r} u$. Our proof relies on a quantitative quenched invariance principle for the corresponding random walk in balanced random environment and a careful analysis of the directed percolation cluster. This result extends Martins Barlow's Harnack's inequality for i.i.d. bond percolation to the directed case. This is joint work with N.Berger, M. Cohen and X. Guo.

Christina Goldschmidt: **The critical random transposition random walk**

Create continuous-time random walk on the symmetric group by successively composing independent transpositions chosen uniformly at random from among the possibilities, at rate $n/2$. The uniform distribution is stationary for this Markov chain. A well-known result of Schramm states that this process undergoes a phase transition: if $t < 1$, the cycles of the permutation at time t are $\mathcal{O}(\log n)$ in size, whereas for $t > 1$, a positive proportion of the numbers $\{1, 2, \ldots, n\}$ are contained in giant cycles, whose relative sizes are distributed approximately as Poisson-Dirichlet$(0, 1)$ (so that although the whole random walk is far from having reached stationarity, it has mixed on part of the space). In this talk, I will characterise the behaviour of the critical random transposition random walk, and shed light on the emergence of the Poisson-Dirichlet distribution. This is joint work with Dominic Yeo.

Enrique Guerra: **A proof of Sznitman's conjecture about ballistic RWRE**

We consider a random walk in a uniformly elliptic i.i.d. random environment in $\mathbb{Z}^d$ for $d \geq 2$. It is believed that whenever the random walk is transient in a given direction it is necessarily ballistic. To some extent, in order to quantify the gap which would be needed to prove this equivalence, several ballisticity conditions have been introduced.

In particular, in [4, 5], Sznitman defined the so called conditions (T) and (T'). The first one is the requirement that certain unlikely exit probabilities from a set of slabs decay exponentially fast with their width L. The second one is the requirement that for all $\gamma \in (0, 1)$ condition $(T)_\gamma$ is satisfied, which in turn is defined as the requirement that the decay is like e^{-CL^γ} for some $C > 0$. These conditions in conjunction with renormalization methods have been used to prove ballistic regime and diffusive scaling limit for the random walk process, even in non-independent settings (cf. [2]).

In this talk we will present a recent result in collaboration with A. F. Ramírez [3] that proves a conjecture of Sznitman of 2002 [5], stating that (T) and (T') are equivalent. Hence, this closes the circle proving the equivalence of conditions (T), (T') and $(T)_\gamma$ for some $\gamma \in (0, 1)$ as conjectured in [5], and also of each of these ballisticity conditions with the polynomial condition $(P)_M$ for $M \geq 15d + 5$ introduced in [1].

[1] N. Berger, A. Drewitz and A.F. Ramírez. *Effective Polynomial Ballisticity Conditions for Random Walk in Random Environment.* Comm. Pure Appl. Math. 67, 1947-1973, (2014).

[2] E. Guerra. *On the transient (T) condition for random walk in mixing environment.* to appear in Ann. Probab. 2019-.

[3] E. Guerra and A.F. Ramírez. *A proof of Sznitman's conjecture about ballistic RWRE.* accepted for publication in Comm. Pure Appl. Math.

[4] A.S. Sznitman. *On a class of transient random walks in random environment.* Ann. Probab. 29, 724-765, (2001).

[5] A.S. Sznitman. *An effective criterion for ballistic behavior of random walks in random environment.* Probab. Theory Related Fields 122, 509-544, (2002).

Xiaoqin Guo: **Quantitative homogenization in a balanced random environment**

Stochastic homogenization of difference operators is closely related to the convergence of random walk in a random environment (RWRE) to its limiting process. In this talk we discuss non-divergence form difference operators in an i.i.d random environment and the corresponding process—a random walk in a balanced random environment in the integer lattice. We first quantify the ergodicity of the environment viewed from the point of view of the particle. As consequences we obtain algebraic rates of convergence for the quenched central limit theorem of the RWRE and for the homogenization of both elliptic and parabolic non-divergence form difference operators. Joint work with J. Peterson (Purdue) and H. V. Tran (UW-Madison).

Jack Hanson: **Universality of the time constant for critical first-passage percolation on the triangular lattice**

We consider first-passage percolation (FPP) on the triangular lattice with vertex weights whose common distribution function F satisfies $F(0) = 1/2$. This is known as the critical case of FPP because large (critical) zero-weight clusters allow travel between distant points in time which is sublinear in the distance. Denoting by T_n the first-passage time from 0 to the boundary of the box of sidelength n, we show existence of the time constant - the limit of $T_n/\log n$ - and find its exact value to be $I/(2\pi\sqrt{3})$. (Here $I = \inf\{x > 0 : F(x) > 1/2\}$.) This shows that the time constant is universal, in the sense that it is insensitive to most details of F. Furthermore, we find the exact value of the limiting normalized variance, which is also only a function of I, under the optimal moment condition on F.

Mark Holmes: **Random graphs arising from reinforcement processes**

Equip each vertex of a graph with an independent Poisson clock. When the clock goes off at a vertex x, choose one of the edges incident to x with probability proportional to the current edge count raised to the power $\alpha > 0$. Add 1 to the count of the chosen edge. Repeat. The set of edges that are chosen infinitely often defines a random graph that has interesting features (depending on the value of α) that we will discuss during this talk. (Based on various joint papers with Victor Kleptsyn, and with VK and Christian Hirsch, and with Remco van der Hofstad, Alexey Kuznetsov, and Wioletta Ruszel).

Daniel Kious: **Random walk on the simple symmetric exclusion process**

In a joint work with Marcelo R. Hilário and Augusto Teixeira, we investigate the long-term behavior of a random walker evolving on top of the simple symmetric exclusion process (SSEP) at equilibrium. At each jump, the random walker is subject to a drift that depends on whether it is sitting on top of a particle or a hole. The asymptotic behavior is expected to depend on the density ρ in $[0, 1]$ of the underlying SSEP. Our first result is a law of large numbers (LLN) for the random walker for all densities ρ except for at most two values ρ_- and ρ_+ in $[0, 1]$, where the speed (as a function fo the density) possibly jumps from, or to, 0.

Second, we prove that, for any density corresponding to a non-zero speed regime, the fluctuations are diffusive and a Central Limit Theorem holds. For the special case in which the density is 1/2 and the jump distribution on an empty site and on an occupied site are symmetric to each other, we prove a LLN with zero limiting speed. Our main results extend to environments given by a family of independent simple symmetric random walks in equilibrium.

Peter Mörters: **The disordered Chinese restaurant and other competing growth processes**

I present a Poisson limit theorem in the framework of competing growth processes with random birth times and random growth rates. I demonstrate applications of these results to preferential attachment graphs as well as a disordered version of the Chinese restaurant process. Based on joint ongoing work with Cecile Mailler and Anna Senkevich (Bath).

Chiranjib Mukherjee: **The KPZ equation in $d \geq 3$ and the Gaussian multiplicative chaos in the Wiener space**

In the classical finite dimensional setting, a Gaussian multiplicative chaos (GMC) is obtained by tilting an ambient measure by the exponential of a centred Gaussian field indexed by a domain in the Euclidean space. In the two-dimensional setting and when the underlying field is "log-correlated", GMC measures share close connection to the 2D Liouville quantum gravity, which has seen a lot of revived interest in the recent years. A natural question is to construct a GMC in the infinite dimensional setting, where techniques based on log-correlated fields in finite dimensions are no longer available. In the present context, we consider a GMC on the classical Wiener space, driven by a (mollified) Gaussian space-time white noise. In $d \geq 3$, in a previous work with A. Shamov and O. Zeitouni, we showed that the total mass of this GMC, which is directly connected to the (smoothened) Kardar-Parisi-Zhang equation in $d \geq 3$, converges for small noise intensity to a well-defined strictly positive random variable, while for larger intensity (i.e. for small temperature) it collapses to zero. We will report on joint work with Yannic Bröker (Münster) where we study the endpoint distribution of a Brownian path under the GMC measure and show that, for low temperature, the endpoint GMC distribution localizes in few spatial islands and produces asymptotically purely atomic states.

Eviatar Procaccia: **Stationary Hastings-Levitov model**

We construct and study a stationary version of the Hastings-Levitov(0) model. We prove that unlike the classical model, in the stationary case particle sizes are constant in expectation, yielding that this model can be seen as a tractable off-lattice Diffusion Limited Aggregation (DLA). The stationary setting together with a geometric interpretation of the harmonic measure yields new geometric results such as stabilization, finiteness of arms and unbounded width in mean of arms. We will present a conjecture for the fractal dimension. Joint work with Noam Berger, Jacob Kagan and Amanda Turner.

Pierre-François Rodriguez: **Around random walks and percolation**

We will discuss recent progress regarding the geometry of some canonical strongly correlated fields in three and more dimensions. In particular, we will present results on the percolation problem for level-sets of the Gaussian free field, first investigated by Lebowitz and Saleur in 1986. We will also describe their links with various fragmentation questions for random walks.

Ron Rosenthal: **Frobenius stability and random groups**

We will discuss the notion of group stability and in particular Frobenius stability and will show that a random group in the triangular model is almost surely Frobenius stable. Based on a work in progress with Lev Glebsky and Alexander Lubotzky.

Artem Sapozhnikov: **Decoupling inequalities for the random walk loop soup**

In this talk we focus on connectivity properties of the random walk loop soup and its vacant set on $\mathbb{Z}^d$ in dimensions $d \geq 3$. We discuss certain decoupling inequalities for the loop soup. As an application, we derive various large scale regularity results for the unique infinite cluster of the loop soup and of the vacant set (e.g., the quenched invariance principle and Gaussian heat kernel bounds for the random walk on the infinite cluster). Joint work with Caio Alves (University of Leipzig).

Vitali Wachtel: **Various constructions of a harmonic function for a random walk in a cone.**

For a random walk killed at leaving a cone we discuss various constructions of a positive harmonic function. These constructions allow one to prove conditional limit theorems for walks in cones which are either convex or star-like and C^2.