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Topology

Problem Set 13

1. (10 points) Let $p, q \in \mathbb{Z}$ and consider the map $f : T^2 \to T^2 = S^1 \times S^1$ defined by $(s,t) \mapsto (s^p, t^q)$, where multiplication is understood as multiplication in $S^1 \subset \mathbb{C}$. Prove that for some identification $H_1(T^2) \cong \mathbb{Z}^2$ and $H_2(T^2) \cong \mathbb{Z}$ the map f_* corresponds to $(m,n) \mapsto (pm, qn)$ on $H_1(T^2)$ and to $m \mapsto pqm$ on $H_2(T^2)$.

   Hint: One possibility is to use the Mayer-Vietoris sequence and naturality of the long exact sequence.

2. (10 points) Let X be a CW-complex and $K \subset X$ a compact subset. Prove that K is contained in a finite subcomplex of X.

3. (10 points) Let X be a CW-complex. Prove that every point has a simply-connected neighbourhood. Also consider the *Hawaiian earring* i.e. the subspace of $\mathbb{R}^2$ which is the union over $n \in \mathbb{N}$ of circles of radius $1/n$ around the center $(1/n, 0)$ and prove that it does not carry a CW-structure.

4. (10 points) Define a CW-complex structure on $\mathbb{R}P^n$ for $n \geq 1$.

Please hand in your solutions on January 28 at the end of the lecture.