Decoherence Through Coupling to the Radiation Field

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Abstract. We consider a quantum particle in a coherent superposition of two well localized wave packets. Through an external harmonic potential the two wave packets are accelerated towards each other, start to overlap, and an interference pattern emerges. When the particle is coupled to the radiation field, assumed to be in a thermal state, the interference pattern is partially suppressed, which defines the decoherence amplitude. For our model we compute both the decoherence and the friction rate. They are given in terms of fundamental constants and a certain scaling function of the temperature. With the exception of unphysical regimes our results are in agreement with the prediction of the master equation for this system.

1 Introduction

Quantum mechanical superpositions are fragile objects and break up easily unless well isolated from the environment. While on an intuitive level this insight goes back to the early days of quantum mechanics, the quantitative study of the process of decoherence dates from the '80 [1, 2, 3, 4, 5]. Usually one considers a "small" quantum system coupled to a suitable environment. Besides the intrinsic unitary time evolution the quantum system is now subject to friction and fluctuating forces. The main conclusion is that, in general, quantum superpositions decohere way before dissipative effects are of any significance.

On a theoretical level the standard set–up is to consider an initial state which is a coherent superposition of two localized wave packets of width σ separated by a distance d with $d \gg \sigma$. One couples this quantum system to a suitable environment and studies how long the superposition is maintained. Two particular types of environments have been investigated in considerable detail

• a free Bose field with massless excitations linearly coupled to the quantum system,

• collisions with a low density gas of massive particles.

To follow the dynamics of such an open quantum system, one popular route is to approximate the environment through a quantum mechanical master equation. While developed originally for a different purpose, master equations have been a very useful guide in unraveling the various facets of decoherence. On the strictly Hamiltonian level, one usually resorts to the free Bose field, since it can be integrated out exactly and leads to a Feynman–Vernon influence functional encoding the information on decoherence. In the dipole approximation and with a harmonic external potential the time evolution can be solved exactly.

In this paper we add to the long list of case studies a harmonically bound particle coupled to the radiation field in the dipole approximation, which is in the class of exactly solved models. Experts have assured us that in the context of decoherence this model has not been considered before. We see two reasons for undertaking the present effort.

(i) The environment is *not* modelled phenomenologically. Coupling parameters and dispersion relations are known and given in terms of fundamental constants.

(ii) The temperature dependence can be studied systematically. In particular, one can follow quantitatively the cross over from coherence to decoherence.

We divide our paper in three parts. In Section 2 we recall the model together with the method of solution and define the particular initial state of interest. In Section 3 we compute friction and decoherence coefficients. In Section 4 we discuss our results and add some general remarks.

2 Particle coupled to the radiation field

We consider a (spinless) quantum particle in three dimensions, mass m, charge $-e$, position $\boldsymbol{q}$, momentum $\boldsymbol{p}$, bound by the external harmonic potential $V(\boldsymbol{q}) = \frac{1}{2}m\omega_0^2\boldsymbol{q}^2$. The particle is coupled to the radiation field quantized in the Coulomb gauge [6]. The dynamical variables are then the two transverse components of the vector potential $\boldsymbol{A}, \nabla\cdot\boldsymbol{A} = 0$, and as canonically conjugate variables the transverse electric field, $-\boldsymbol{E}_\perp$. In the momentum representation one introduces the two component Bose field $a(\boldsymbol{k},j), a^\dagger(\boldsymbol{k},j), j = 1,2$, $\boldsymbol{k}\in\mathbb{R}^3$, with commutation relations $[a(\boldsymbol{k},j), a^\dagger(\boldsymbol{k}',j')] = \delta_{jj'}\delta(\boldsymbol{k}-\boldsymbol{k}')$. Let $\boldsymbol{e}_1(\boldsymbol{k}), \boldsymbol{e}_2(\boldsymbol{k}), \boldsymbol{k}/|\boldsymbol{k}|$ form a left–handed dreibein. The transverse part of the vector potential is then given by

$$\boldsymbol{A}(\boldsymbol{x}) = \sum_{j=1}^{2}\int d^3k\boldsymbol{e}_j(\boldsymbol{k})\frac{\hbar}{\sqrt{\hbar\omega(\boldsymbol{k})}}(2\pi)^{-3/2}\,\frac{1}{\sqrt{2}}[e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}a^\dagger(\boldsymbol{k},j) + e^{i\boldsymbol{k}\cdot\boldsymbol{x}}a(\boldsymbol{k},j)] \quad (2.1)$$

and the transverse part of the electric field by

$$\boldsymbol{E}_\perp(\boldsymbol{x}) = \sum_{j=1}^{2}\int d^3k\boldsymbol{e}_j(\boldsymbol{k})\sqrt{\hbar\omega(\boldsymbol{k})}(2\pi)^{-3/2}\frac{1}{i\sqrt{2}}[e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}a^\dagger(\boldsymbol{k},j) - e^{i\boldsymbol{k}\cdot\boldsymbol{x}}a(\boldsymbol{k},j)]\,. \quad (2.2)$$

Here $\omega(\boldsymbol{k}) = c|\boldsymbol{k}|$ is the dispersion relation of the radiation field. The Hamiltonian of the photon field reads

$$\begin{aligned} H_f &= \frac{1}{2}:\int d^3x(c^2|\nabla\times\boldsymbol{A}(\boldsymbol{x})|^2 + |\boldsymbol{E}_\perp(\boldsymbol{x})|^2): \\ &= \sum_{j=1}^{2}\int d^3k\hbar\omega(\boldsymbol{k})a^\dagger(\boldsymbol{k},j)\,a(\boldsymbol{k},j)\,. \end{aligned} \quad (2.3)$$

The particle is minimally coupled to the radiation field, i.e. as $(\boldsymbol{p}+e\,\boldsymbol{A}(\boldsymbol{q}))^2/2m$. In the dipole approximation one replaces $\boldsymbol{A}(\boldsymbol{q})$ by $\boldsymbol{A}(0)$, $\boldsymbol{q} = 0$ being the center of the external potential. By (2.1), $\boldsymbol{A}(0)$ is an operator on Fock space. To have it well defined one needs that the coupling coefficients are square integrable, i.e. $\int d^3k\omega(\boldsymbol{k})^{-1} < \infty$. Clearly, this integral diverges at high wave numbers and we have to introduce an ultraviolet cut–off. In position space this corresponds to smearing as $\boldsymbol{A}(\rho) = \int d^3x \rho(\boldsymbol{x})\, \boldsymbol{A}(\boldsymbol{x})$, where $\hat{\rho}(\boldsymbol{k}) = (2\pi)^{-3/2} \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \rho(\boldsymbol{x})$ with $\hat{\rho}(\boldsymbol{k}) = (2\pi)^{-3/2}$ for $|\boldsymbol{k}| \leq k_{max}$ and $\hat{\rho}(\boldsymbol{k}) = 0$ for $|\boldsymbol{k}| \geq k_{max}$. Alternatively, one could introduce a smooth cut–off. We will see that our results depend only little on the cut–off, as long as it is within a physically reasonable range. Thus we arrive at the coupled Hamiltonian

$$H = \frac{1}{2m}(\boldsymbol{p} \otimes 1 + e 1 \otimes \boldsymbol{A}(\rho))^2 + V(\boldsymbol{q}) \otimes 1 + 1 \otimes H_f\,. \tag{2.4}$$

The Hilbert space of states is $L^2(\mathbb{R}^3) \otimes (\mathcal{F}_b \oplus \mathcal{F}_b)$ with $\mathcal{F}_b$ the bosonic Fock space over the one–particle space $L^2(\mathbb{R}^3)$. States evolve in time according to the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi_t = H\psi_t\,.$$

The Heisenberg equations of motion are linear and coincide with the classical evolution equations, which we write down for completeness. We denote the classical variables again as $(\boldsymbol{q}, \boldsymbol{p}, \hat{\boldsymbol{A}}(\boldsymbol{k}), -\hat{\boldsymbol{E}}_\perp(\boldsymbol{k}))$, without risk of confusion. They evolve as

$$\begin{aligned}
\frac{d}{dt}\boldsymbol{q}(t) &= \frac{1}{m}(\boldsymbol{p}(t) + e\int d^3k' \hat{\rho}(\boldsymbol{k}')\, Q(\boldsymbol{k}')\, \hat{\boldsymbol{A}}(\boldsymbol{k}', t))\,, \\
\frac{d}{dt}\boldsymbol{p}(t) &= -m\,\omega_0^2 \boldsymbol{q}(t)\,, \\
\frac{\partial}{\partial t}\hat{\boldsymbol{A}}(\boldsymbol{k}, t) &= -\hat{\boldsymbol{E}}_\perp(\boldsymbol{k}, t)\,, \\
\frac{\partial}{\partial t}\hat{\boldsymbol{E}}_\perp(\boldsymbol{k}, t) &= \omega(k)^2 \hat{\boldsymbol{A}}(\boldsymbol{k}, t) \\
&+ e\hat{\rho}(\boldsymbol{k}) Q(\boldsymbol{k}) \frac{1}{m}(\boldsymbol{p}(t) + e\int d^3k' \hat{\rho}(\boldsymbol{k}') Q(\boldsymbol{k}') \hat{\boldsymbol{A}}(\boldsymbol{k}', t))\,,
\end{aligned} \tag{2.5}$$

where $Q(\boldsymbol{k})\boldsymbol{u} = \sum_{j=1}^{2} e_j(\boldsymbol{k})(e_j(\boldsymbol{k}) \cdot \boldsymbol{u})$ is the transverse projection. Through the solution of (2.5) a Heisenberg operator at time t is expressed as a linear combination of the Heisenberg operators at time $t = 0$. E.g. for the position operator we have

$$\boldsymbol{q}(t) = c(t)\boldsymbol{q} + b(t)\boldsymbol{p} + e\int d^3k \hat{\rho}(\boldsymbol{k}) Q(\boldsymbol{k})[-\dot{g}_{\boldsymbol{k}}(t)\hat{\boldsymbol{A}}(\boldsymbol{k}) + g_{\boldsymbol{k}}(t)\hat{\boldsymbol{E}}_\perp(\boldsymbol{k})]\,. \tag{2.6}$$

The coefficients $c(t), b(t), g_{\boldsymbol{k}}(t)$ come from the solution to (2.5), while in (2.6) $\boldsymbol{q}, \boldsymbol{p}, \hat{\boldsymbol{A}}(\boldsymbol{k}), \hat{\boldsymbol{E}}_\perp(\boldsymbol{k})$ are time zero operators as defined above in (2.1), (2.2). They will have to be properly averaged over the initial quantum state.

We assume that initially the particle is in a coherent superposition of two Gaussian wave packets centered at $\pm\boldsymbol{d} = (\pm d, 0, 0)$. Let φ be such a wave packet centered at 0 with $\langle\varphi|\varphi\rangle = 1$, $\langle\varphi|\boldsymbol{q}|\varphi\rangle = 0$, $\langle\varphi|q_i^2|\varphi\rangle = \sigma^2$, $\langle\varphi|\boldsymbol{p}|\varphi\rangle = 0$, $\langle\varphi|p_i^2|\varphi\rangle = (\hbar/2\sigma)^2$, $i = 1, 2, 3$. Then the normalized coherent superposition is

$$\psi = \langle\psi_+ + \psi_-|\psi_+ + \psi_-\rangle^{-1/2}(\psi_+ + \psi_-)\,, \quad \psi_\pm(\boldsymbol{x}) = \varphi(\boldsymbol{x} \mp \boldsymbol{d})\,. \tag{2.7}$$

The photon field is assumed to be in the thermal state at inverse temperature $\beta = 1/k_B T$ with statistical operator $\rho_\beta = Z^{-1}\exp[-\beta H_f]$. The initial density matrix of the coupled system is then

$$\rho = (|\psi\rangle\langle\psi|) \otimes \rho_\beta\,. \tag{2.8}$$

Let us set $e = 0$ for a moment. Then ψ_+, ψ_- are accelerated towards the origin. At time $t = \pi/2\omega_0 = \tau$ they interfere maximally with a pattern of wave number $\boldsymbol{k}_0 = (2dm\omega_0/\hbar, 0, 0)$. For *non–zero* coupling we have

$$\mathrm{tr}[\boldsymbol{q}(t)(|\psi_+\rangle\langle\psi_+| \otimes \rho_\beta)] = \boldsymbol{d}\, e^{-\gamma t}\cos\tilde{\omega}_0 t \tag{2.9}$$

with exponentially small relative corrections. (2.9) defines the friction coefficient γ. We will assume that

$$\gamma\tau = \gamma\pi/2\omega_0 \ll 1\,. \tag{2.10}$$

We are allowed then to ignore the frequency shift to $\tilde{\omega}_0$, i.e. $\tilde{\omega}_0 \cong \omega_0$, and the two parts of the wave packet still maximally overlap at time $t = \tau$. The interference pattern has a slightly shifted wave number. However the amplitude is damped by the factor

$$e^{-\Gamma\tau} \tag{2.11}$$

which defines decoherence coefficient Γ. We will compute γ and Γ for our model in the following section.

3 Friction and decoherence coefficients

We are interested in the time of first maximal interference, i.e. $t = \pi/2\omega_0 = \tau$, more precisely $\mathrm{tr}[q_1(\tau)(|\psi_+\rangle\langle\psi_+| \otimes \rho_\beta)] = \langle q_1(\tau)\rangle_{++} = 0$. Of course one could carry out the computation for arbitrary times, but there is no need.

We discuss friction first, which is visible in $\langle \boldsymbol{q}(t)\rangle_{++}$. Since $\langle \boldsymbol{p}\rangle_{++} = 0$, $\langle \hat{\boldsymbol{A}}(\boldsymbol{k})\rangle_{++} = 0$, $\langle \hat{\boldsymbol{E}}_\perp(\boldsymbol{k})\rangle_{++} = 0$, we only have to solve the classical equations of motion with initial condition $(\boldsymbol{d}, 0, 0, 0)$ and obtain

$$\int_0^\infty dt\, e^{-zt} \langle \boldsymbol{q}(t)\rangle_{++} = \boldsymbol{d}\, z(z^2 + \omega_0^2\,(1 + \ell(z))^{-1})^{-1}\,, \tag{3.1}$$

where

$$\ell(z) = (e^2/m)\,\frac{2}{3}\int d^3k |\hat{\rho}(\boldsymbol{k})|^2\,(z^2 + \omega(\boldsymbol{k})^2)^{-1}\,. \tag{3.2}$$

For $e = 0$, the Laplace transform (3.1) has the two poles at $z_\pm = \pm i\,\omega_0$. For small e these poles wander to the left, their (negative) real part being the friction coefficient. Possibly, there are other singularities moving in from $-\infty$, but for small e their contribution is negligible compared to the poles at $z_\pm$ [7]. To lowest order the friction coefficient

$$\gamma = \frac{1}{6\pi}\,\frac{e^2}{\hbar c}\,\frac{\hbar\omega_0}{mc^2}\,\omega_0\,. \tag{3.3}$$

We require $\gamma\tau \ll 1$, i.e.

$$\frac{1}{12}\,\frac{e^2}{\hbar c}\,\frac{\hbar\omega_0}{mc^2} \quad \ll 1\,. \tag{3.4}$$

The decoherence coefficient requires more work. Since the motion is only along the 1–axis, we set $x_1 = y$ to simplify notation. We define

$$\rho_{ij} = (|\psi_i\rangle\langle\psi_j|) \otimes \rho_\beta\,, \quad i,j = +,-\,. \tag{3.5}$$

$\rho_{++}, \rho_{+-}, \rho_{-+}, \rho_{--}$ are Gaussian density matrices and they remain Gaussian under the unitary time evolution. Since we are interested only in the spatial interference pattern, we need the distribution of q_1 with respect to $\rho_{\pm\pm}(t)$ which we denote by $\tilde{\rho}_{\pm\pm}(y,t)$. They are Gaussian and therefore determined by their first and second moments. We set

$$\begin{aligned}
\langle q_1(t)\rangle_{\pm\pm} &= \mathrm{tr}[\rho_{\pm\pm}(t) q_1]/\mathrm{tr}[\rho_{\pm\pm}] = \int dy\, \tilde{\rho}_{\pm\pm}(y,t) y = a_{\pm\pm}(t),\\
\langle q_1(t)^2\rangle_{\pm\pm} &= \mathrm{tr}[\rho_{\pm\pm}(t)\, q_1^2]/\mathrm{tr}[\rho_{\pm\pm}] = \int dy \tilde{\rho}_{\pm\pm}(y,t) y^2\,,\\
\Delta_{\pm\pm}(t) &= \langle q_1(t)^2\rangle_{\pm\pm} - \langle q_1(t)\rangle_{\pm\pm}^2\,.
\end{aligned} \tag{3.6}$$

Then at time $t = \tau$

$$
\begin{aligned}
&\langle \psi_+ + \psi_- | \psi_+ + \psi_- \rangle^{-1} \left(\tilde{\rho}_{++}(y,t) + \tilde{\rho}_{+-}(y,t) + \tilde{\rho}_{-+}(y,t) + \tilde{\rho}_{--}(y,t) \right) \\
&= \langle \psi_+ + \psi_- | \psi_+ + \psi_- \rangle^{-1} 2 \,(2\pi\Delta)^{-1/2}\, e^{-y^2/2\Delta} \\
&[1 + \langle \psi_+ | \psi_- \rangle \, \exp\left[-(a^2/2\Delta)\right] \cos(iay/\Delta)] \qquad (3.7)
\end{aligned}
$$

with $\Delta = \Delta_{++}(\tau)$ and $a = a_{+-}(\tau)$. Clearly

$$
e^{-\Gamma\tau} = \langle \psi_+ | \psi_- \rangle \, \exp[-(a^2/2\Delta)] \,. \qquad (3.8)
$$

We emphasize that (3.8) is valid only for Gaussian wave packets.

Since $\langle p_1 \rangle_{+-} = id\hbar/2\sigma^2$, we have

$$
a = (id\hbar/2\sigma^2) \; (1/\omega_0 m) \,. \qquad (3.9)
$$

To compute Δ we use (2.6). Then

$$
\begin{aligned}
\Delta &= \langle q_1(\tau)^2 \rangle_{++} - \langle q_1(\tau) \rangle_{++}^2 = b(\tau)^2 \; \langle p_1^2 \rangle_{++} \\
&\quad + e^2 \langle [\int d^3k \hat{\rho}(\boldsymbol{k}) \, Q(\boldsymbol{k}) (-\dot{g}_{\boldsymbol{k}}(\tau) \hat{\boldsymbol{A}}(\boldsymbol{k}) + g_{\boldsymbol{k}}(\tau) \hat{\boldsymbol{E}}_\perp(\boldsymbol{k}))]^2 \rangle_{++} \,. \qquad (3.10)
\end{aligned}
$$

¿From above $\langle \varphi | p_i^2 | \varphi \rangle = (\hbar/2\sigma)^2$ and, since friction is small by assumption, we have $b(\tau) = 1/m\,\omega_0$. To obtain the coefficients $g_{\boldsymbol{k}}(t)$, we need the solution $\boldsymbol{q}(t)$ of the classical equations of motion for general initial data. As before, their Laplace transform is explicit [7]. Since friction is small, we set approximately $\ell(z) = 0$. Then the Laplace transform can be inverted with the result

$$
g_{\boldsymbol{k}}(t) = -\frac{1}{m\omega_0} \int_0^t ds \sin\omega_0(t-s) \; \cos\omega(\boldsymbol{k})s \qquad (3.11)
$$

and therefore

$$
\begin{aligned}
g_{\boldsymbol{k}}(\tau) &= -(1/m\omega_0^2) \int_0^{\pi/2} ds \cos s \; \cos(\omega(\boldsymbol{k})s/\omega_0) \\
\dot{g}_{\boldsymbol{k}}(\tau) &= (1/m\omega_0) \int_0^{\pi/2} ds \sin s \; \cos(\omega(\boldsymbol{k})s/\omega_0) \,. \qquad (3.12)
\end{aligned}
$$

Finally we need

$$
\langle \psi_+ | \psi_- \rangle = \exp\left[-d^2/2\sigma^2\,\right] . \qquad (3.13)
$$

Inserting in (3.8) we obtain

$$\Gamma\tau = \frac{1}{2}\,(\frac{d}{\sigma})^2\,\frac{\delta}{1+\delta} \tag{3.14}$$

with the dimensionless factor

$$\delta = (2\sigma m\omega_0/\hbar)^2 e^2 \langle [\int d^3k \hat{\rho}(\boldsymbol{k})\, Q(\boldsymbol{k})\,(-\dot{g}_{\boldsymbol{k}}(\tau)\,\hat{\boldsymbol{A}}(\boldsymbol{k}) + g_{\boldsymbol{k}}(\tau)\hat{\boldsymbol{E}}_\perp(\boldsymbol{k}))]^2\rangle_{++}\,. \tag{3.15}$$

To compute the thermal average we use (2.1), (2.2) and

$$\langle a^\dagger(\boldsymbol{k},j)\,a(\boldsymbol{k}',j')\rangle_{++} = \delta_{jj'}\delta(\boldsymbol{k}-\boldsymbol{k}')\,(e^{\beta\hbar\omega(\boldsymbol{k})}-1)^{-1}\,. \tag{3.16}$$

Then

$$e^2\langle[\cdot]^2\rangle_{++} = \hbar\, e^2 (m\omega_0)^{-2}\int d^3k |\hat{\rho}(\boldsymbol{k})|^2 \omega(\boldsymbol{k})^{-1}\coth\left(\beta\hbar\omega(\boldsymbol{k})/2\right) \tag{3.17}$$
$$\{(\int_0^{\pi/2} ds \sin s \cos(\omega(\boldsymbol{k})s/\omega_0))^2 + (\omega(\boldsymbol{k})/\omega_0)^2 (\int_0^{\pi/2} ds \cos s \cos(\omega(\boldsymbol{k})s/\omega_0))^2\}.$$

Therefore

$$\delta = \frac{e^2}{\hbar c}\,(2\sigma\omega_0/c)^2\, f(\beta\hbar\omega_0) \tag{3.18}$$

with the scaling function

$$f(u) = \frac{1}{2\pi^2}\int_0^{(k_{\max}c/\omega_0)} dw\, w \coth(wu/2)\{(1-w^2)^{-2}(1-2w\sin(\pi w/2)+w^2)\}. \tag{3.19}$$

The function in the curly brackets equals 1 at $w=0$, is finite at $w=1$, and decays as w^{-2} for large w. Thus the integral in (3.19) is logarithmically divergent at the upper border.

4 Discussion of the results

The condition of small friction is

$$\frac{e^2}{\hbar c}\,\frac{\hbar\omega_0}{mc^2} < 1\,, \tag{4.1}$$

cf. (3.4). For decoherence we distinguish two regimes. If

$$\delta > 1\,, \quad \text{then} \quad \Gamma\tau = \frac{1}{2}\,(d/\sigma)^2\,. \tag{4.2}$$

As seen from the specific example below, $\delta > 1$ requires an unphysically high temperature and we can safely disregard this regime. If

$$\delta < 1\,, \quad \text{then} \quad \Gamma\tau = \frac{1}{2}\frac{e^2}{\hbar c}\,(2d\omega_0/c)^2 f(\beta\hbar\omega_0)\,. \tag{4.3}$$

In this regime the decoherence is independent of the width σ, as long as $\delta \ll 1$ is satisfied. If $\beta\hbar\omega_0 < 1$, then

$$\begin{aligned} f(\beta\hbar\omega_0) \;&\cong\; \frac{1}{\beta\hbar\omega_0}\frac{1}{\pi^2}\int_0^{(k_{max}c/\omega_0)} dw\{(1-w^2)^{-2}(1-2w\sin(\pi w/2)+w^2)\} \\ &\cong\; \frac{1}{\beta\hbar\omega_0} \end{aligned} \tag{4.4}$$

independent of the cut–off. On the other hand if $\beta\hbar\omega_0 > 1$, then

$$\begin{aligned} f(\beta\hbar\omega_0) \;&\cong\; \frac{2}{\pi^2}\int_0^{(k_{max}c/\omega_0)} dww\{(1-w^2)^{-2}(1-2w\sin(\pi w/2)+w^2)\} \\ &\cong\; \log(k_{max}c/\omega_0)\,. \end{aligned} \tag{4.5}$$

Thus $f(u)$ crosses over to a constant at $u = u_0$ which depends on the choice of the cut–off. Again from the example below, if one sets the cross over scale by $\beta\hbar\omega_0 = 1$, one is already deeply in the coherence regime. Physically the cut–off dependent piece of $f(\beta\hbar\omega_0)$ can be ignored. Thus, up to the clauses discussed, we have the final result

$$\Gamma\tau = \frac{1}{2}\frac{e^2}{\hbar c}(2d\omega_0/c)^2\,\frac{1}{\beta\hbar\omega_0}\,, \tag{4.6}$$

where the integral in (4.4) has been set equal to one, approximately.

In the literature, decoherence is often expressed relative to friction, i.e. as Γ/γ, which in our model is given by

$$\frac{\Gamma}{\gamma} = 48\pi\Big(\frac{d}{\lambda_{th}}\Big)^2\,, \quad \lambda_{th}^2 = 2\pi\hbar^2/mk_{\mathrm{B}}T\,, \tag{4.7}$$

in agreement with results from master equations and from the influence functional [4]. Of course, in our model both Γ and γ is computed separately. In addition, we believe to have followed a more physical approach. We define the friction by slowing down and the decoherence through the amplitude of the interference pattern, whereas usually only the off-diagonal pieces of the density matrix in the position representaton are discussed [5].

To have a feeling for the order of magnitudes involved we compute the various coefficients for an electron. We choose $d = 10^{-3}m$, $\sigma = 0.5\text{Å}$, the Bohr radius, and require an interference pattern with a wave length $\lambda_0 = 100\text{Å}$.

The friction coefficient is

$$\gamma\tau = 2 \times 10^{-17} \tag{4.8}$$

an (4.1) holds easily. To have interference with wave length λ_0, the frequency of the oscillator

$$\omega_0 = 3 \times 10^7/\sec\,, \quad \hbar\omega_0 = 2 \times 10^{-8}\text{eV}\,. \tag{4.9}$$

$\delta > 1$ corresponds to

$$\beta\hbar\omega_0 < 7 \times 10^{-25} \ll 1\,. \tag{4.10}$$

$\beta\hbar\omega_0 = 7 \times 10^{-25}$ is equivalent to a temperature $T = 3 \times 10^{20}\,^\circ K$, which is way beyond the physical validity of our model. If $\delta < 1$ and $\beta\hbar\omega_0 \ll 1$, Eq. (4.6) holds with

$$\Gamma\tau = 1.4 \times 10^{-8}/\beta\hbar\omega_0\,. \tag{4.11}$$

If $\beta\hbar\omega_0 = 1$, we have already $\Gamma\tau = 10^{-8}$ and the cut–off dependent regime is not accessible. We define the cross over temperature, T_d, from coherence to decoherence by $\Gamma\tau = 1$. In our example

$$T_d = 2 \times 10^4\,^\circ K\,. \tag{4.12}$$

We recall that $\Gamma\tau$ enters in the exponent. Thus as a function of the temperature the cross over from coherence to decoherence is very sharp.

We add some general remarks.

(i) If there is *no* external potential and we choose ψ_+, ψ_- such that their average momenta are pointing towards each other, then at temperature $T = 0$ physically one would expect perfect interference no matter how large the coherence length d. While this is correctly reproduced by letting $T \to 0$ in the first regime, the

dipole approximation looses its validity when d is too large and one should use the standard translation invariant coupling. This is a more difficult task, since the corresponding Hamiltonian can no longer be diagonalized exactly.
(ii) Decoherence results from two very distinct mechanisms, namely thermal noise and emission of real photons because of acceleration. In the dipole approximation $1/\beta$ is the strength of the thermal noise and $\hbar\omega_0$ reflects the acceleration. From (4.6) we see that $\Gamma\tau$ is proportional to ω_0/β, i.e. both effects enter linearly.
(iii) As exemplified by our computation decoherence manifests itself already after a short time, e.g. one oscillation or, in a collision model, one collision time. However in most studies, including our own, on a theoretical level this is in fact not well exploited. It would be of interest to develop tools for a quantitative investigation of decoherence in physically realistic models using the unitary evolution over short times only.

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